2015

Assessment Task 4
Trial HSC Examination

Mathematics Extension 1

Examiners ~ Mrs D. Crancher, Mrs S. Gutesa, Mr S. Faulds, Ms P. Biczo

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen.
- Board approved calculators and mathematical templates and instruments may be used.
- Show all necessary working in Questions 11,12,13 and 14
- This examination booklet consists of 13 pages including a standard integral page and a multiple choice answer sheet.

Total marks (70)

Section I

Total marks (**10**)

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided on the last page of this question booklet.
- Allow about 15 minutes for this section

Section II

Total marks (**60**)

- Attempt questions11 to 14
- Answer each question in the writing booklets provided.
- Start a new booklet for each question with your student name and question number at the top of the page.
- All necessary working should be shown for every question
- Allow about 1 hour 45 minutes for this section

Student Name : ______________________________

Teacher : ______________________________

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

---

1 The solution to the inequality $x(2-x)(x+1) \geq 0$ is

(A) $x \leq -2$ or $0 \leq x \leq 1$ (B) $-2 \leq x \leq 0$ or $x \geq 1$

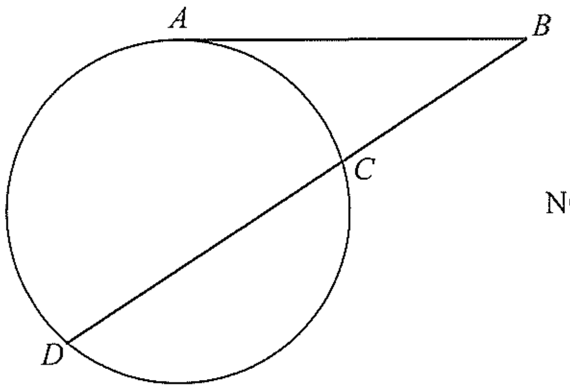
(C) $x \leq -1$ or $0 \leq x \leq 2$ (D) $-1 \leq x \leq 0$ or $x \geq 2$

2 A committee of 3 men and 3 women is to be formed from a group of 8 men and 6 women. How many ways can this be done?

(A) 48 (B) 1120

(C) 40320 (D) 3003

3



NOT TO SCALE

In the diagram, AB is a tangent to the circle, $BC = 6\text{cm}$ and $CD = 12\text{cm}$.
What is the length of AB?

(A) $6\sqrt{2}$ cm (B) $6\sqrt{3}$ cm

(C) 72cm (D) 108cm

4 What is the equation of the tangent at the point $(4p, 2p^2)$ on the parabola $x^2 = 8y$?

(A) $y = px - p^2$ (B) $x + py = 2p + p^3$

(C) $x + py = 4p + p^3$ (D) $y = px - 2p^2$

5 What is the acute angle to the nearest degree that the line $2x-3y+5=0$ makes with the y-axis?

(A) 27° (B) 34°

(C) 56° (D) 63°

6 Which of the following statements is FALSE.

(A) $\cos^{-1}(-\theta)=-\cos^{-1}\theta$ (B) $\sin^{-1}(-\theta)=-\sin^{-1}\theta$

(C) $\tan^{-1}(-\theta)=-\tan^{-1}\theta$ (D) $\cos^{-1}(-\theta)=\pi-\cos^{-1}\theta$

7 The primitive of $2x\left(3x^2-1\right)^4$ is:

(A) $\frac{1}{15}\left(3x^2-1\right)^5+c$ (B) $\frac{3}{5}\left(3x^2-1\right)^5+c$

(C) $\frac{2x}{5}\left(3x^2-1\right)^5+c$ (D) $\frac{2x}{15}\left(3x^2-1\right)^5+c$

8 The equation(s) of the horizontal asymptote(s) to the curve $y=\frac{x^2+1}{x^2-1}$ are

(A) $y=0$ (B) $x=\pm1$

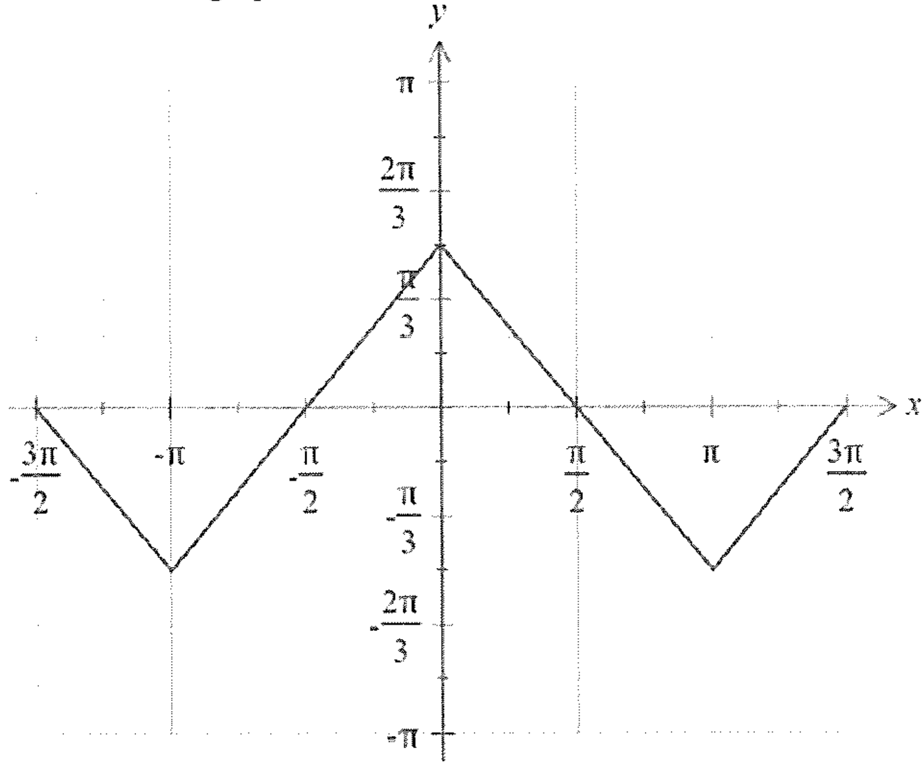
(C) $y=1$ (D) $x=1$ only

9 What are the coordinates of the point that divides the interval joining the points $A(2,2)$ and $B(4,5)$ externally in the ratio 2:3?

(A) $(-2,-4)$ (B) $(-2,11)$

(C) $(8,-4)$ (D) $(8,11)$

10 Which of the following equations is shown in the sketch below



(A) $y = \cos^{-1}(\sin x)$ (B) $y = \sin^{-1}(\cos x)$

(C) $y = \sin^{-1}(x) + \sin(x)$ (D) $y = \cos^{-1}(x) + \cos(x)$

~ End of Section I ~

Section II

60 marks
Attempt Questions 11 to 14

Allow about 1 hour 45 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Marks**

(a) Solve the inequality $\dfrac{3}{x(2x-1)} > 1$? **3**

(b) In what ratio does the point $(14,18)$ divide the interval joining $X\ (-1,3)$ to $Y\ (4,8)$? **2**

(c) (i) Show that the curves $y = x^3 - x$ and $y = x - x^2$ intersect at the point $(-2,-6)$ **1**

(ii) Determine the acute angle between the curves $y = x^3 - x$ and $y = x - x^2$ at the point of intersection, to the nearest minute. **3**

(d) (i) A class of 25 students is to be divided into four groups consisting of 3, 4, 5 and 6 students. How many ways can this are done? Leave your answer in unsimplified form. **2**

(ii) Assume that the four groups have been chosen.
How many ways can the 25 students be arranged around a circular table if the students in each group are to be seated together?
Leave your answer in unsimplified form. **2**

(e) Five different fair dice are thrown together.
What is the probability the five scores are all different? **2**

Question 12 (15 marks) **Marks**

(a) Consider the function $f(x) = (x-1)^2$

(i) Sketch $y = f(x)$. **1**

(ii) Explain why $f(x)$ does not have an inverse function for all x in its domain. **1**

(iii) State a domain and range for which $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(iv) For $x \geq 1$ find the equation of the function $f^{-1}(x)$. **2**

(v) Hence , on a new set of axes, sketch the graph of $y = f^{-1}(x)$. **1**

(b) Find $\displaystyle\int \frac{dx}{\sqrt{9-4x^2}}$ **2**

(c) Find the exact value of $\tan\left(2\tan^{-1}\frac{3}{4}\right)$ **2**

(d) Find the general solution to $2\cos x = \sqrt{3}$.
Leave your answer in terms of π. **2**

(e) Differentiate (with respect to x)

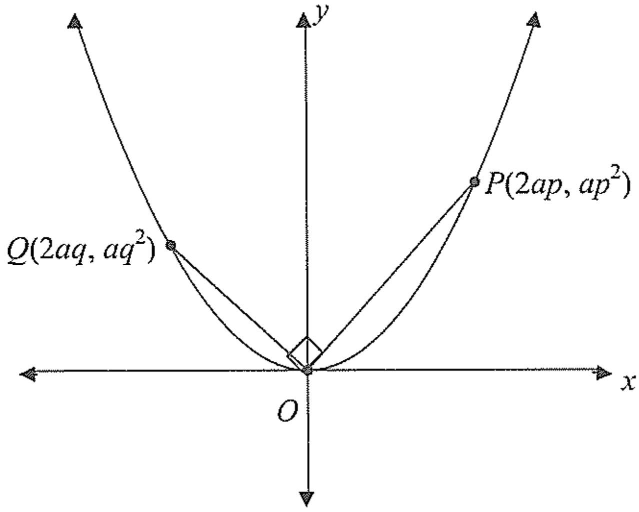
$$\left(\tan^{-1}\frac{x}{3}\right)^2$$

and hence find the exact value of

$$\int_0^{\sqrt{3}} \frac{\tan^{-1}\frac{x}{3}}{x^2+9}\,dx$$ **3**

Question 13 (15 marks) **Marks**

(a) The points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$ such that OP is perpendicular to OQ.



(i) Prove that $pq = -4$. **2**

(ii) R is the point such that $OPRQ$ is a rectangle.
Explain why the co-ordinates of R are $\left(2a(p+q),\, a\left(p^2+q^2\right)\right)$. **2**

(iii) Show that the locus of R is a parabola. **2**

(b) Find by division of polynomials, the remainder when x^2+4 is divided by $x-3$. **1**

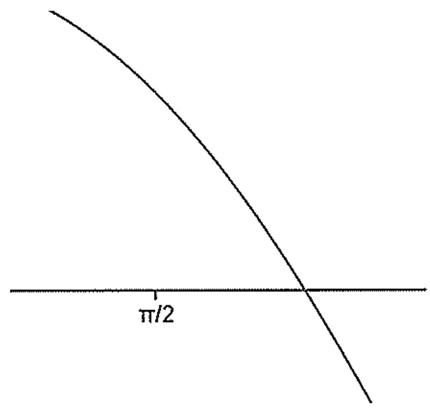
(c) α, β and γ are the roots of the equation $x^3-3x^2-6x-1=0$.
Find $\alpha^2+\beta^2+\gamma^2$. **2**

Question 13 continued next page....

(d) (i) Consider the curve $f(x) = \sin^2 x - x + 1$ for $0 \le x \le \pi$.
Show that it has one stationary point and determine its nature. **3**

(ii) $f(x) = \sin^2 x - x + 1$ has a zero near $x_1 = \dfrac{\pi}{2}$.
Use one application of Newton's method to obtain another approximation x_2, to this zero. **2**

(iii)

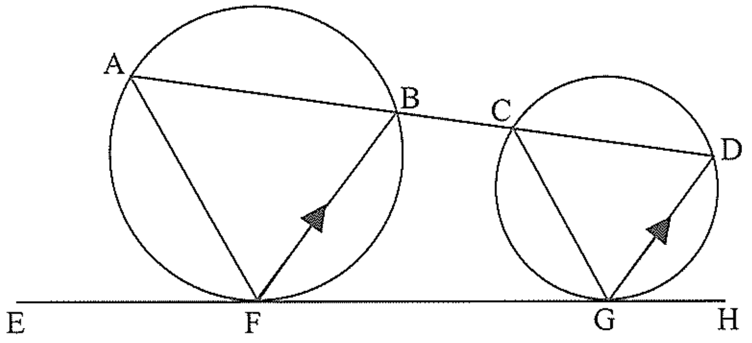


The graph of $f\left(x\right) = \sin^2 x - x + 1$ is shown in the vicinity of $x = \dfrac{\pi}{2}$.

By using this diagram, determine if x_2 is a better approximation than x_1 to the real root of the equation. You must justify your answer. **1**

Question 14 (15 marks) **Marks**

(a)



In the diagram above, FG is a common tangent and FB||GD.

(i) Prove that FA||GC. **2**

(ii) Prove that BCGF is a cyclic quadrilateral. **2**

(b) (i) Find: $\frac{d}{dx}(x\sin 3x)$. **2**

(ii) Hence, evaluate: $\int_0^{\frac{\pi}{6}} x\cos 3x\,dx$ **3**

(c) Use the substitution $y=\sqrt{x}$ to find

$$\int \frac{dx}{\sqrt{x(1-x)}}$$ **3**

(d) Use mathematical induction to prove the inequality: **3**

$n! > 2^n$, for all positive integral values of $n \geq 4$

~ End of Section II ~

Year 12 Mathematics Extension 1 Trial 2015

Question No. 11 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3 Solves problems involving permutations and combinations, inequalities and polynomials.
H5 Applies appropriate techniques from the study of geometry.

Outcome	Solutions	Marking Guidelines
PE3	**(a)**	**3 marks** Correct solution **2 marks** Substantial progress towards correct solution **1 mark** Some progress towards correct solution

$$\frac{3}{x(2x-1)} > 1$$

Multiply by the square of the denominator

$$3x(2x-1) > x^2(2x-1)^2$$

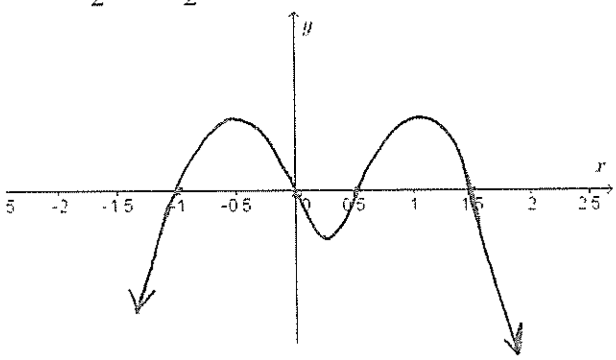
$$3x(2x-1) - x^2(2x-1)^2 > 0$$

$$x(2x-1)(3 - x(2x-1)) > 0$$

$$x(2x-1)(-2x^2 + x + 3) > 0$$

$$-x(2x-1)(2x-3)(x+1) > 0$$

$$\therefore -1 < x < 0,\ \ \frac{1}{2} < x < \frac{3}{2}$$



Outcome	Solutions	Marking Guidelines
H5	**(b)**	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.

$A(-1,3)\quad B(4,8)\quad P(14,18)$

Ratio $AB : BP$, $m:n$ assume $1:k$

$$x_0 = \frac{mx_2 + nx_1}{m+n}$$

$$14 = \frac{1(4) + k(-1)}{1+k}$$

$$14 + 14k = 4 - k$$

$$k = \frac{-2}{3}$$

$$1 : k = 1 : \frac{-2}{3}$$

$\therefore$ Ratio is $-3:2$

Outcome	Solutions	Marking Guidelines
H5	**(c)(i)**	**1 mark** Correct solution

Substitute $x = -2$ into $y = x^3 - x$

$$RHS = (-2)^3 - (-2)$$
$$= -6$$

$\therefore$ Satisfies the curve.

Substitute $x = -2$ into $y = x - x^2$

$$RHS = -2 - (-2)^2$$
$$= -6$$

$\therefore$ Satisfies the curve.

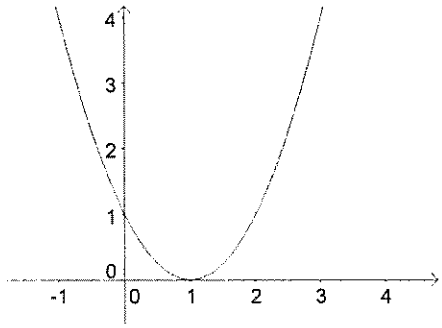
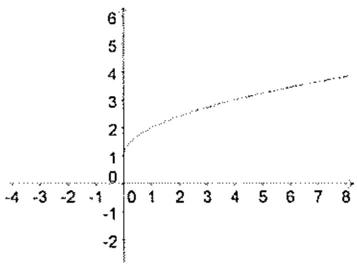
$\therefore (-2,-6)$ is the point of intersection.

	(c)(ii) For $y = x^3 - x$, $y' = 3x^2 - 1$ when $x = -2$, $m_1 = 3(-2)^2 - 1 = 11$ For $y = x - x^2$, $y' = 1 - 2x$ when $x = -2$, $m_2 = 1 - 2(-2) = 5$ $\tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ $\tan\theta = \left\|\frac{11-5}{1+(11)(5)}\right\|$ $\therefore \theta = \tan^{-1}\left(\frac{6}{56}\right)$ $\therefore \theta = 6°7'$ (to the nearest minute)	**3 marks** Correct solution with correct rounding **2 marks** Substantial progress towards correct solution **1 mark** Some progress towards correct solution
PE3	**(d) (i)** ${}^{25}C_3 \times {}^{22}C_4 \times {}^{18}C_5 \times {}^{13}C_6$ or $\frac{25!}{3!\,4!\,5!\,6!\,7!}$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution
PE3	**(d) (ii)** $(11-1)! \times 3! \times 4! \times 5! \times 6!$	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.
H5	**(e)** $P(E) = \frac{6\times5\times4\times3\times2}{6^5}$ $= \frac{5}{54}$	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.

Year 12 2015	Mathematics Extension 1	Task 4 Trial HSC
Question No. 12	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

HE4 uses the relationship between functions, inverse functions and their derivatives

Outcome	Solutions	Marking Guidelines
HE4	Question 12 a) (i) 	1 Mark for correct sketch
HE4	(ii) It does not have an inverse because for every y value there is more than one x value. Or Does not pass the horizontal line test. Or Anything that is equivalent.	1 Mark for correct explanation
HE4	(iii) Domain: $x \geq 1$ Range: $y \geq 0$	1 Mark for correct answer
HE4	(iv) $x=(y-1)^2$ $\sqrt{x}=y-1$ $y=1+\sqrt{x}$ $\therefore f^{-1}(x)=1+\sqrt{x}$	2 Marks for complete correct solution 1 Mark for partial correct solution
HE4	(v) 	1 Mark for correct sketch
HE4	(b) $\int \frac{dx}{\sqrt{9-4x}}$ $=\int \frac{dx}{\sqrt{4\left(\frac{9}{4}-x^2\right)}}$ $=\frac{1}{2}\sin^{-1}\left(\frac{x}{\frac{3}{2}}\right)+C$ or $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+C$	2 Marks for complete correct solution 1 Mark for partial correct solution

HE4	(c) let $\theta = \tan^{-1}\left(\frac{3}{4}\right) \quad \therefore \tan\theta = \frac{3}{4}$ now, $\tan\left(2\tan^{-1}\frac{3}{4}\right) = \tan(2\theta)$ $= \frac{2\tan\theta}{1-\tan^2\theta}$ $= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$ $= \frac{24}{7}$	2 Marks for complete correct solution 1 Mark for partial correct solution
HE4	(d) $2\cos x = \sqrt{3}$ $\cos x = \frac{\sqrt{3}}{2}$ $\therefore x = 2n\pi \pm \cos^{-1}\frac{\sqrt{3}}{2}$ $= 2n\pi \pm \frac{\pi}{6}$, where n is any integer.	2 Marks for complete correct solution 1 Mark for partial correct solution
HE4	(e) $\frac{d}{dx}\left(\tan^{-1}\frac{x}{3}\right)^2 = 2\left(\tan^{-1}\frac{x}{3}\right)\left(\frac{\frac{1}{3}}{1+\frac{x^2}{9}}\right)$ $= 2\left(\tan^{-1}\frac{x}{3}\right)\left(\frac{3}{9+x^2}\right)$ $= 6\left(\frac{\tan^{-1}\frac{x}{3}}{9+x^2}\right)$ now, $\int_0^{\sqrt{3}}\left(\frac{\tan^{-1}\frac{x}{3}}{9+x^2}\right)dx = \frac{1}{6}\int_0^{\sqrt{3}} 6\left(\frac{\tan^{-1}\frac{x}{3}}{9+x^2}\right)dx$ $= \frac{1}{6}\left[\left(\tan^{-1}\frac{x}{3}\right)^2\right]_0^{\sqrt{3}}$ $= \frac{1}{6}\left(\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)^2 - \left(\tan^{-1}\frac{0}{3}\right)^2\right)$ $= \frac{1}{6}\left(\left(\frac{\pi}{6}\right)^2 - 0\right)$ $= \frac{\pi^2}{216}$	3 Marks for complete correct solution 2 Mark for substantial working that could lead to a correct solution with only one error. 1 Mark for correctly differentiating $\frac{d}{dx}\left(\tan^{-1}\frac{x}{3}\right)^2$

Year 12 Trial Higher School Certificate	Extension 1 Mathematics	Examination 2015
Question No. 13	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

PE3 solves problems involving polynomials and parametric representations
PE5 determines derivatives which require the application of more than one rule of differentiation
H6 uses the derivative to determine the features of the graph of a function
HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form.

Outcome	**Solutions**	**Marking Guidelines**
PE3	(a) (i) $OP \perp OQ$, $\therefore\ mOP \times \text{m}\, OQ = -1$. $\therefore \dfrac{ap^2}{2ap} \times \dfrac{aq^2}{2aq} = -1$ $\therefore \dfrac{p}{2} \times \dfrac{q}{2} = -1$ $\therefore pq = -4$.	2 marks : correct solution 1 mark : significant progress towards correct solution
PE3	(ii) Midpoint $PQ = \left(ap + aq, \dfrac{ap^2 + aq^2}{2} \right)$ As the diagonals bisect one another in a rectangle, OR will also have the same midpoint as PQ. If $O(0,0)$, R has midpoint $\left(ap + aq, \dfrac{ap^2 + aq^2}{2} \right)$, then R is $\left(2a(p+q), a\left(p^2 + q^2\right) \right)$.	2 marks : correct solution 1 mark : significant progress towards correct solution
PE3	(iii) At R, $\begin{cases} pq = -4 & [1] \\ x = 2a(p+q) & [2] \\ y = a\left(p^2+q^2\right) & [3] \end{cases}$ From $[2]$, $x^2 = 4a^2(p+q)^2$ $\therefore x^2 = 4a^2\left(p^2 + q^2 + 2pq\right)$ Substituting $[1]$ and $[3]$, $x^2 = 4a^2\left(\dfrac{y}{a} + 2 \times -4\right)$, $\therefore x^2 = 4a(y - 8a)$, which is a concave up parabola with vertex $(0, 8a)$.	2 marks : correct solution 1 mark : significant progress towards correct solution
PE3	(b) $x+3$ $x-3\overline{)x^2 + 0x + 4}$ − $\underline{x^2 - 3x}$ $3x + 4$ − $\underline{3x - 9}$ 13 Remainder is 13.	2 marks : correct solution 1 mark : significant progress towards correct solution
	(c) From $x^3 - 3x^2 - 6x - 1 = 0$, $\alpha + \beta + \gamma = \dfrac{-b}{a} = 3,\ \alpha\beta + \beta\gamma + \alpha\lambda = \dfrac{c}{a} = -6$. $(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \alpha\gamma)$	2 marks : correct solution 1 mark : significant progress towards correct solution

$$\therefore \alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)$$

$$\therefore \alpha^2+\beta^2+\gamma^2=(3)^2-2(-6)$$

$$=21.$$

H6, PE5

(d) (i) $f(x)=\sin^2 x-x+1$

$f'(x)=2\sin x\cos x-1$

$\therefore f'(x)=\sin 2x-1$

$f'(x)=0$ for stationary points.

Solving $\sin 2x-1=0,$

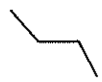
$\sin 2x=1$

For $0\le x\le \pi,\ 0\le 2x\le 2\pi.$

Solving, $2x=\dfrac{\pi}{2}$, $\therefore$ one stationary point, at $x=\dfrac{\pi}{4}$.

Testing $x=\dfrac{\pi}{4}$, for $f'(x)=\sin 2x-1$,

x	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$f'(x)$	$\frac{\sqrt{3}-2}{2}$	0	$\frac{\sqrt{3}-2}{2}$



As $\dfrac{\sqrt{3}-2}{2}$ is negative, there is a horizontal point of inflexion at $x=\dfrac{\pi}{4}$.

3 marks : correct solution
2 marks : substantial progress towards correct solution
1 mark: significant progress towards correct solution

PE5, HE7

(ii) $f(\frac{\pi}{2})=\sin^2\frac{\pi}{2}-\frac{\pi}{2}+1=2-\frac{\pi}{2}$

$f'\left(\frac{\pi}{2}\right)=\sin\pi-1=-1.$

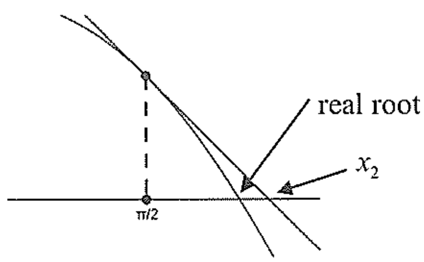
Newton's method : $x_2=\dfrac{\pi}{2}-\dfrac{2-\frac{\pi}{2}}{-1}$

$\therefore x_2=2$

2 marks : correct solution
1 mark : significant progress towards correct solution

HE7

(iii)



x_2 is where the tangent at $\dfrac{\pi}{2}$ meets the x axis.

This is closer to the real root, $\therefore$ a better approximation.

1 mark : correct explanation

Year 12 Mathematics Extension 1 Trial Examination 2015

Question No. 14 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3 solves problems involving circle geometry
HE2 uses inductive reasoning in the construction of proofs
HE4 uses the relationship between functions, inverse functions and their derivatives
HE6 determines integrals by reduction to a standard form through a given substitution

Outcome	Solutions	Marking Guidelines
PE3	**(a)(i)** Let $\angle DGH = \alpha$ $\therefore \angle BFG = \alpha$ (corresponding angles, FB‖GD) Now, $\angle GCD = \alpha$ (angle between a chord and tangent is equal to the angle in the alternate segment) Similarly, $\angle FAB = \alpha$ Since $\angle GCD = \angle FAB = \alpha$, $FA \parallel GC$ (corresponding angles are equal)	**2 marks** Correct solution with full reasoning. **1 mark** Substantial progress towards a correct solution.
PE3	**(ii)** Since $\angle GCD = \alpha$ (shown above) $\angle GCB = 180° - \alpha$ (angles on a straight line also, $\angle BFG = \alpha$ (shown above) $\angle GCB + \angle BFG = 180° - \alpha + \alpha$ $= 180°$ $\therefore$ BCGF is a cyclic quadrilateral (opposite angles supplementary)	**2 marks** Correct solution with full reasoning. **1 mark** Substantial progress towards a correct solution.
HE4	**(b) (i)** $\frac{d}{dx} x\sin 3x = x.3\cos 3x + \sin 3x.1$ $= 3x\cos 3x + \sin 3x$	**2 marks** Correct application of product rule to find correct answer. **1 mark** Demonstrates knowledge of product rule in making substantial progress to a full solution.
HE4	**(ii)** If $\frac{d}{dx} x\sin 3x = 3x\cos 3x + \sin 3x$ then $3x\cos 3x = \frac{d}{dx} x\sin 3x - \sin 3x$ $x\cos 3x = \frac{1}{3}\frac{d}{dx} x\sin 3x - \frac{1}{3}\sin 3x$ Integrating both sides, $\int_0^{\frac{\pi}{6}} x\cos 3x dx = \frac{1}{3}\int_0^{\frac{\pi}{6}} \frac{d}{dx} x\sin 3x dx - \frac{1}{3}\int_0^{\frac{\pi}{6}} \sin 3x dx$ $= \frac{1}{3}\left[x\sin 3x + \frac{1}{3}\cos 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{1}{3}\left[\left(\frac{\pi}{6}\sin\frac{\pi}{2} + \frac{1}{3}\cos\frac{\pi}{2}\right) - \left(0\sin 0 + \frac{1}{3}\cos 0\right)\right]$ $= \frac{1}{3}\left[\left(\frac{\pi}{6} - 0\right) - \left(0 + \frac{1}{3}\right)\right]$ $= \frac{1}{3}\left(\frac{\pi}{6} - \frac{1}{3}\right)$ $= \frac{\pi - 2}{18}$	**3 marks** Correct solution. **2 marks** Correctly finds the required primitive function. **1 mark** Substantial progress towards finding the required primitive function.

HE6	**(c)** Let $y=\sqrt{x}$ $\therefore x=y^2$ $\frac{dx}{dy}=2y$ $dx=2ydy$ $\int\frac{dx}{\sqrt{x(1-x)}}=\int\frac{2ydy}{\sqrt{y^2(1-y^2)}}$ $=\int\frac{2ydy}{y\sqrt{(1-y^2)}}$ $=2\int\frac{dy}{\sqrt{(1-y^2)}}$ $=2\sin^{-1}y+c$ but $y=\sqrt{x}$ $\therefore\int\frac{dx}{\sqrt{x(1-x)}}=2\sin^{-1}\sqrt{x}+c$	**3 marks** Correct solution. **2 marks** Uses the given substitution correctly and makes substantial progress towards a correct solution. **1 mark** Uses the given substitution correctly.
HE2	**(d)** $n!>2^n$, for all positive integral values of $n\geq 4$ Prove true for $n=4$ $LHS=4!$ $\quad$ $RHS=2^4$ $=24$ $\quad$ $=16$ $24>16$ $\therefore$ True for $n=4$ Assume true for $n=k$ ie. Assume $k!>2^k$ $k!-2^k>0$ Prove true for $n=k+1$ ie. Prove $(k+1)!>2^{k+1}$ Consider the difference $(k+1)!-2^{k+1}=(k+1).k!-2.2^k$ $=k.k!+k!-2^k-2^k$ $=k.k!-2^k+k!-2^k$ $=(k-1).k!+k!-2^k+k!-2^k$ $=(k-1).k!+2(k!-2^k)$ Now, since $k>4, (k-1)>0$ $k!>0$ $\therefore(k-1).k!>0$ Also, $k!-2^k>0$, from the assumption Hence, $(k-1).k!+2(k!-2^k)>0$ $(k+1)!-2^{k+1}>0$ $\therefore(k+1)!>2^{k+1}$ $\therefore$ By the process of mathematical induction, $n!>2^n$ is true for all positive integral values of $n\geq 4$	**3 marks** Correct solution. **2 marks** Prove the relationship is true for n=4 and makes substantial progress towards a correct solution. **1 mark** Correctly proves the relationship true for n=4.